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OSCILLATORY AND POWER-LAW MASS INFLATION IN NON-ABELIAN BLACK HOLES

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Interior structure of non-Abelian black holes is shown to exhibit in a general case either an oscillating mass-inflationary behavior, or power-law behavior with a divergent mass function. In both cases no Cauchy horizon forms.

Mass inflation inside black holes emerges as back reaction on the perturbations caused by the cross-flow of radiation tails in the vicinity of the Cauchy horizon (CH).[1] A particularly simple picture of this effect arises in the case of homogeneous (i.e., t–independent) perturbations in spherical black holes. Such a situation may be also treated non-linearly as an interior problem for a static black hole in the framework of a suitable Einstein–matter theory. An interesting example is provided by the Einstein–Yang–Mills (EYM) system,[2] or its extensions including scalar fields: dilaton [4] or Higgs.[3] In the first theory an internal Cauchy horizon may exist only for a discrete sequence of black hole masses. For a generic mass the true CH is not formed, but, when such a 'would be' CH is approached, the mass function starts to grow exponentially. However, the singularity is not formed instead of CH to the contrary to the perturbative prediction. In the full non-linear treatment this local mass inflation stops shortly, and the metric relaxes to the next 'would be' CH. Then the process is repeated again resulting in an oscillatory behavior of the mass function with an infinitely growing amplitude. It is remarkable that maximal values of the mass function attained in subsequent cycles also increase exponentially, so globally we deal with a kind of a 'quantized' mass inflation. The ultimate singularity is spacelike and is not power-law.[5]

Spherical EYM black holes are described by a single YM function $W(r)$, and by two metric functions $\Delta = r^2/g_{rr}$ and $\sigma^2 = g_{tt}g_{rr}$, where r is a two-sphere radius. When Δ approaches zero (being negative) inside the black hole, at some $r = r_k$ the derivative $W' = dW/dr$ becomes approximately a linear function of r, $W' = rU_k$, with an almost constant U_k. Then the behavior of Δ is governed with a good accuracy by the equation

$$(\Delta/r)' + 2\Delta U^2 = 0,$$

which gives locally

$$\Delta(r) = \frac{\Delta(r_k)}{r_k}\, r\, \exp\left[U_k^2(r_k^2 - r^2)\right]. \tag{1}$$

This function falls down exponentially with decreasing r until it reaches a local minimum at $R_k = 1/(\sqrt{2}|U_k|)$. The mass function $m(r)$ ($\Delta = r^2 - 2mr$) therefore is exponentially inflating when one moves leftward from r_k to R_k. The function W stabilizes near the limiting value although W' may be very large at some tiny intervals. The corresponding behavior of σ follows from an approximate integral $Z = \Delta U\sigma/r = \text{const}$, which is valid throughout the oscillation region:

$$\sigma(r) = \sigma(r_k)\exp\left[U_k^2(r^2 - r_k^2)\right].$$

After passing R_k, an exponential in (1) becomes of the order of unity, hence Δ grows linearly, and the mass function $m(r)$ stabilizes at the value $M_k = m(R_k)$. Such a behavior holds until the point of the local maximum of Δ/r^2, which takes place when $\Delta \approx -V^2$ ($V = W^2 - 1$) at the point

$$r_k^* \approx \frac{V^2}{|\Delta(r_k)|} r_k \exp\left[-(U_k r_k)^2\right].$$

After that a rapid fall of $|\Delta|$ is observed causing a violent rise of $|U|$ according to $U\Delta \approx -V^2 U_k$, while r remains practically constant. Then Δ reaches the next local maximum at the point $r_{k+1} \approx r_k^*$, while $m(r)$ rapidly falls down to m_{k+1}.

This sequence can be described by the order of magnitude in terms of the following exponentially diverging sequence:

$$x_{k+1} = x_k^{-3} \mathrm{e}^{x_k},$$

where $x_k = (r_k/R_k)^2$ ($\gg 1$). In terms of x_k one has

$$\frac{r_{k+1}}{r_k} = x_k \mathrm{e}^{-x_k/2}, \quad |\Delta(r_k)| = x_k^{-1},$$

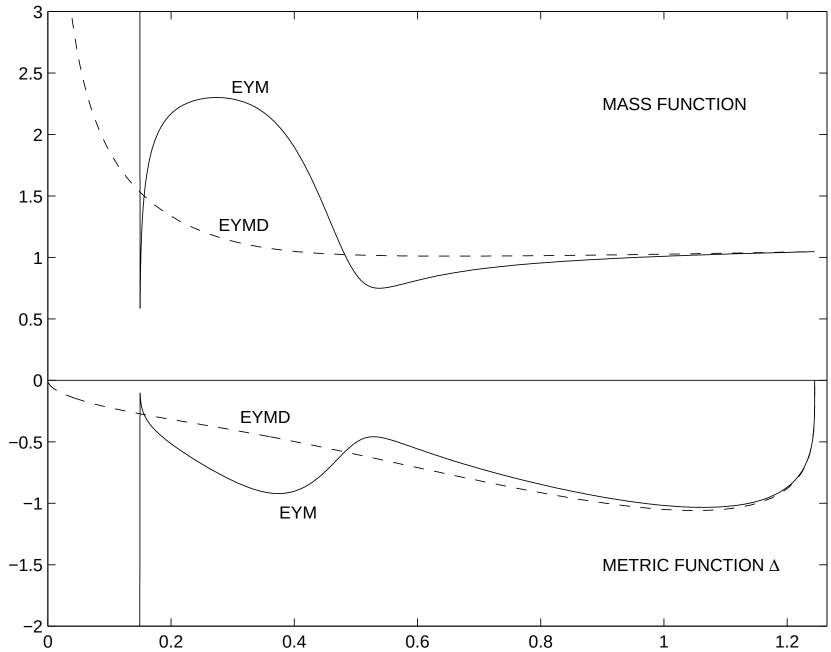
so we deal with an infinite sequence of "almost" Cauchy horizons as $r \to 0$. At the same time the values of $|\Delta|$ and m at R_k grow rapidly

$$|\Delta(R_k)| = x_k^{-3/2}\mathrm{e}^{x_k/2}, \quad \frac{M_k}{M_{k-1}} = x_k^{-1}\mathrm{e}^{x_k/2}.$$

Finally, $\sigma_{k+1}/\sigma_k = \mathrm{e}^{-x_k/2}$.

The same theory with dilaton [4] predicts qualitatively different behavior of generic black hole solutions in the interior region. In this case after some oscillations the mass function is attracted to a monotonic power-law solution without CH-s, which terminates in a spacelike singularity. Similar behavior is observed in the EYM models with doublet or triplet Higgs.[3] In both cases the attractor solution is described by the truncated system in which kinetic scalar terms are dominant:

$$(\ln U)' - 2\phi' = 0, \quad [\ln(\Delta/r)]' = [\ln(\Delta\phi')]' = -r\phi'^2.$$



Its integration gives the following five-parameter, i.e., generic family of solutions

$$W = W_0 + br^{2(1-\lambda)}, \quad \Delta = -2\mu r^{(1-\lambda^2)}, \quad \phi = c + \ln\left(r^{-\lambda}\right), \tag{2}$$

with constant W_0, b, c, μ, λ. Parameter λ is subject to some restrictions, which ensure scalar dominance. They differ in the EYMD and EYMH cases.[4, 3] It follows from (2) that the mass function diverges as $r \to 0$ according to the power law $m(r) = \mu r^{-\lambda^2}$. The corresponding σ tends to zero as $\sigma(r) = \sigma_1 r^{\lambda^2}$, where $\sigma_1 =$ const.

This behavior seems to be rather general for the theories, which include scalar fields. We term it power-law mass inflation. No exponential mass inflation is observed in such theories since the metric does not approach to the internal CH at all. So, in a sense, power-law mass inflation provides an alternative to the standard mass inflation scenario. Both types of the mass and metric functions behavior for the EYM and EYMD cases are shown in the figure (coordinate r and all the functions are given in power 1/4).

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1. E. Poisson and W. Israel, *Phys. Rev. Lett.* **63**, 1663 (1989); *Phys. Rev.* D **41**, 1976 (1990); A. Ori, *Phys. Rev. Lett.* **67**, 789 (1991).
2. E.E. Donets, D.V. Gal'tsov, and M.Yu. Zotov, *Internal structure of Einstein–Yang–Mills black holes*, gr-qc/9612067, *Phys. Rev.* D **56**, 3459 (1997).
3. D.V. Gal'tsov and E.E. Donets, *Power-law mass inflation in Einstein–Yang–Mills–Higgs black holes*, gr-qc/9706067.

4. D.V. Gal'tsov, E.E. Donets, and M.Yu. Zotov, *Pis'ma Zh. Eksp. Teor. Fiz.* **65**, 855 (1997), [*JETP Lett.* **65**, 895 (1997)]; (gr-qc/9706063).
5. D.V. Gal'tsov, E.E. Donets, and M.Yu. Zotov, *Singularities inside hairy black holes*, this volume.